

# Analytic Solutions of Brane in Critical Gravity

Yu-Xiao Liu<sup>1,2\*</sup>, Yong-Qiang Wang<sup>1†</sup>, Shao-Feng Wu<sup>3‡</sup>, and Yuan Zhong<sup>1§</sup>

<sup>1</sup>*Institute of Theoretical Physics, Lanzhou University, Lanzhou 730000, China*

<sup>2</sup>*Interdisciplinary Center for Theoretical Study, University of Science and Technology of China, Hefei, Anhui 230026, China*

<sup>3</sup>*Department of Physics, Shanghai University, Shanghai 200444, China*

Recently, H. Lu and C.N Pope et al proposed critical gravities (quadratic-curvature actions with cosmological constant) in four and higher dimensions. At the critical point, these theories possess such an AdS vacuum, for which there is only massless tensor, and the linearized excitations have vanishing energy. In this paper we construct analytic braneworld solutions in critical gravities with matter in diverse dimensions. Both thin and thick branes with co-dimension one are considered. It is found that at the critical point the equations of motion (EOMs) are second-order, and the thin and thick brane solutions are obtained. The curvature-squared modifications in the four-dimensional critical gravity do not affect the brane solutions, but they will do in higher dimensions. All these branes are embedded in higher-dimensional AdS spacetimes.

PACS numbers:

It has been known that, by combining the Einstein-Hilbert action higher derivative terms such as Ricci and scalar curvature squared terms, power-counting renormalizable theories of gravity can be realized. In the absence of the cosmological term, although the theory is renormalizable, it suffers from having ghosts and is perturbatively non-unitary [1, 2].

Recently, motivated by the works of chiral topologically massive gravity with negative cosmological constant in three dimensions [3, 4], critical gravities (quadratic-curvature actions with cosmological constant) in four and higher dimensions has been constructed [5, 6]. At the critical point, these theories possess such an AdS vacuum, for which there is only massless tensor, and the linearized excitations have vanishing energy. It was also shown that at the critical point the theory admits additional modes, the so-called logarithmic modes [5, 7–9], which arise as limits of the massive spin 2 modes of the non-critical theory [9]. The quantization of the linear fluctuations of these critical gravities was studied in Ref. [10] and it has been pointed out that there may be a trouble with the unitarity. So, ghost modes should be eliminated by boundary conditions. The condition that the theory is unitary and stable for four-dimensional critical gravity was analyzed in Ref. [11].

So far, the known background solutions for critical gravity theories are AdS vacuum solutions. And the analysis of metric perturbations is based on the AdS vacuum or the black holes in the AdS background [5, 6, 10, 12]. This characteristic of critical gravity naturally reminds us with the following question: does critical gravity support Randall-Sundrum (RS) braneworld solution? It is well-known that RS braneworld model offers us a solu-

tion to the hierarchy problem by embedding two 3-branes in an AdS<sub>5</sub> spacetime [13, 14]. But in the original set up, the gravity is described by general relativity, which is non-renormalizable. Thus it is a natural idea to reconstruct RS braneworld model in some renormalizable gravity theories. Another important question is to seek analytic non-AdS solutions in these critical gravity theories. These analytical solutions can help us to study the stability of critical gravity in non-AdS spacetime explicitly. However, both of these attempts are non-trivial, because the equations of motion in generalized gravity theory are fourth-order in general.

In this paper, we would like to search for braneworld solutions in critical gravities in diverse dimensions. Both RS thin and thick branes with co-dimension one are considered. It is found that at the critical point the equations of motion are second-order, and so the thin branes can be realized. For simplicity we embed only a single brane, the generalization to multi-branes can be done and will be illustrated in our future works.

First, we consider the thin braneworld with co-dimension one generated in  $n$ -dimensional critical gravity, where  $n \geq 4$ . The action is

$$S = S_g + S_b, \quad (1)$$

where the gravity part  $S_g$  and the brane part  $S_b$  are given by

$$S_g = \frac{1}{2\kappa^2} \int d^n x \sqrt{-g} \left[ R - (n-2)\Lambda_0 + \alpha R^2 + \beta R_{MN} R^{MN} \right], \quad (2)$$

$$S_b = \int d^{n-1} x \sqrt{-g^{(b)}} (-V_0). \quad (3)$$

Here  $g_{\mu\nu}^{(b)}$  is the induced metric on the brane, and  $V_0$  is the brane tension. The capitals  $M, N, \dots = 0, 1, 2, \dots, n-2, n$  and the Greek letters  $\mu, \nu, \dots = 0, 1, 2, \dots, n-2$  denote the indices of the  $n$ -dimensional bulk and the  $(n-1)$ -dimensional braneworld, respectively. The line-element describing a static flat brane can be assumed as

$$ds^2 = g_{MN} dx^M dx^N = e^{2A(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2, \quad (4)$$

\*liuyx@lzu.edu.cn, corresponding author

†yqwang@lzu.edu.cn

‡sfwu@shu.edu.cn

§zhongy2009@lzu.edu.cn

where  $e^{2A}$  is the warp factor with the normalized condition  $e^{2A(0)} = 1$  on the brane located at  $y = 0$ . We introduce the  $Z_2$  symmetry by setting  $A(y) = A(-y)$ .

The equations of motion are given by

$$\mathcal{G}_{MN} + \alpha E_{MN}^{(1)} + \beta E_{MN}^{(2)} = -\kappa^2 V_0 \delta_M^\mu \delta_N^\nu g_{\mu\nu} \delta(y), \quad (5)$$

where

$$\mathcal{G}_{MN} = R_{MN} - \frac{1}{2} R g_{MN} + \frac{1}{2} (n-2) \Lambda_0 g_{MN},$$

$$E_{MN}^{(1)} = 2R(R_{MN} - \frac{1}{4} R g_{MN}) + 2g_{MN} \square R - 2\nabla_M \nabla_N R,$$

$$E_{MN}^{(2)} = 2R^{PQ}(R_{MPNQ} - \frac{1}{4} R_{PQ} g_{MN}) + \square(R_{MN} + \frac{1}{2} R g_{MN}) - \nabla_M \nabla_N R.$$

The junction conditions are determined by

$$\int_{0^-}^{0^+} dy [\mathcal{G}_{\mu\nu} + \alpha E_{\mu\nu}^{(1)} + \beta E_{\mu\nu}^{(2)}] = -\kappa^2 V_0 g_{\mu\nu}(0). \quad (6)$$

It is very difficult to find the solution of thin brane for arbitrary  $\alpha$  and  $\beta$  for the fourth-order differential equations (5) and the junction conditions (6). However, at the critical point  $4(n-1)\alpha + n\beta = 0$  [5, 6], the EOMs at  $y \neq 0$  are reduced to the following second-order ones:

$$4\Lambda_0 + 4(n-1)A'^2 + (n-1)(n-2)(n-4)\beta A'^4 = 0, \quad (7)$$

$$[2 + (n-2)(n-4)\beta A'^2]A'' = 0, \quad (8)$$

and the junction condition reads

$$\int_{0^-}^{0^+} dy [2 + (n-2)(n-4)\beta A'^2]A'' = -\frac{2\kappa^2}{n-2} V_0, \quad (9)$$

where the prime stands for the derivative with respect to  $y$ . We note from Eqs. (7) and (8) that the curvature-squared modifications in the four-dimensional critical gravity ( $n = 4$ ) have no effect on the brane solutions. In the following, we will give the solutions of the above brane equations for  $n = 4$  and  $n > 4$ , respectively.

For the case  $n = 4$ , the terms containing  $\beta$  in Eqs. (7) and (8) and the junction condition (9) vanish. The solution is

$$A(y) = -\sqrt{-\frac{\Lambda_0}{3}} |y|, \quad (n = 4) \quad (10)$$

and the brane tension and the bulk cosmological are related by

$$V_0 = \frac{4}{\kappa^2} \sqrt{-\frac{\Lambda_0}{3}}. \quad (11)$$

Thus, we get a brane with positive tension and the warp factor exponentially falling from the brane to infinity. The brane is embedded in a four-dimensional AdS spacetime. This is nothing but the RS solution in four dimensions. However, it is worth to note that, although the

curvature-squared modifications in the four-dimensional critical gravity have no effect on the RS thin brane solution, the fluctuation equations of the brane solution in the critical gravity are very different from those in the standard Einstein gravity.

For the case  $n \neq 4$ , we first give the solution corresponding to  $\beta = 0$ :

$$A^{(RS)}(y) = -\sqrt{\frac{-\Lambda_0}{n-1}} |y|, \quad (n > 4, \beta = 0) \quad (12)$$

$$V_0^{(RS)} = \frac{2(n-2)}{\kappa^2} \sqrt{\frac{-\Lambda_0}{n-1}}, \quad (13)$$

which describes the RS positive tension brane embedded in an AdS<sub>n</sub> spacetime.

In critical gravity with nonvanishing  $\beta$ , we have two solutions:

$$A_{\pm}(y) = -\sqrt{\frac{2[\pm\sqrt{\zeta} - (n-1)]}{n_3\beta}} |y|, \quad (n > 4) \quad (14)$$

$$V_{0\pm} = \pm \frac{1}{\kappa^2} \frac{(n-2)}{(n-1)} \sqrt{\frac{-8\Lambda_0\zeta}{(n-1) \pm \sqrt{\zeta}}}. \quad (15)$$

where  $n_3 = (n-1)(n-2)(n-4)$  and  $\zeta = (n-1)^2 - n_3\beta\Lambda_0$ . The subscripts “+” and “-” correspond to positive and negative tension branes, respectively. For the positive tension brane solution, the constrain conditions for the parameters are  $\Lambda_0 < 0$  and  $\beta > 0$ , or  $\Lambda_0 < 0$  and  $\frac{n-1}{(n-2)(n-4)\Lambda_0} < \beta < 0$ . For the negative tension case, the constrain conditions are  $\beta < 0$  and  $\Lambda_0 > 0$ , or  $\beta < 0$  and  $\frac{n-1}{(n-2)(n-4)\beta} < \Lambda_0 < 0$ .

Now, we study the limits of the solutions (14) under the condition of  $\beta \rightarrow 0$ . For the negative tension brane solution  $A_-(y)$ , the limit is divergent. While, for the case of positive tension,  $A_+(y)$  and  $V_{0+}$  can be expanded as

$$A_+(y) = -\sqrt{\frac{-\Lambda_0}{n-1}} [1 + \mathcal{O}(\beta)] |y|, \quad (16)$$

$$V_{0+} = \frac{2(n-2)}{\kappa^2} \sqrt{\frac{-\Lambda_0}{n-1}} [1 + \mathcal{O}(\beta)]. \quad (17)$$

So, when  $\beta \rightarrow 0$ , the above positive tension brane solution (14)-(15) can be reduced to the RS one (12)-(13), while the negative one cannot.

It is interesting to note that, when  $\Lambda_0$  and  $\beta$  satisfy the following relation

$$\Lambda_0 = \frac{n-1}{(n-2)(n-4)\beta}, \quad (18)$$

the brane tension (15) vanishes and the warp factor is simplified as

$$A(y) = -\sqrt{\frac{-2\Lambda_0}{n-1}} |y|. \quad (19)$$

Obviously, such solution could not appear in standard Einstein gravity theory. While, in critical gravity theory,

although the naked brane tension is zero, we can identify  $-\alpha E_{\mu\nu}^{(1)} - \beta E_{\mu\nu}^{(2)}$  as  $\kappa^2 T_{\mu\nu}^{(\text{eff})}$  to get an effective positive brane tension.

Next, we consider the thick brane generated by a scalar field in  $n$ -dimensional critical gravity. The action reads as

$$S = S_g + S_m, \quad (20)$$

where  $S_g$  is given by (2) and the matter part is

$$S_m = \int d^n x \sqrt{-g} \left[ -\frac{1}{2} g^{MN} \partial_M \phi \partial_N \phi - V(\phi) \right]. \quad (21)$$

The cosmological constant  $\Lambda_0$  can be absorbed into the scalar potential. The line-element is also assumed as (4) and the scalar field  $\phi = \phi(y)$  for static branes.

The EOMs for general  $\alpha$  and  $\beta$  are fourth-order, while they are reduced to the following second-order ones in critical case:

$$[(n-2)(n-4)\beta A'^2 - 2] A'' = \frac{2\kappa^2}{n-2} \phi'^2, \quad (22)$$

$$n_3 \beta A'^4 + 4(n-1)A'^2 + 4\Lambda_0 = \frac{8\kappa^2}{n-2} \left( \frac{1}{2} \phi'^2 - V \right), \quad (23)$$

$$\phi'' + (n-1)A'\phi' = V_\phi, \quad (24)$$

where  $V_\phi \equiv \frac{dV}{d\phi}$ . Note that eq. (24) can be derived from eqs. (22) and (23). Hence, the above three equations are not independent.

So, in order to solve the above second-order differential equations, we can use the superpotential method. Introducing the superpotential function  $W(\phi)$ , the EOMs (22)-(24) can be solved by the first-order equations:

$$A' = -\frac{\kappa^2}{n-2} W, \quad (25)$$

$$\phi' = (1 - c_1 W^2) W_\phi, \quad (26)$$

$$V = \frac{1}{2} (1 - c_1 W^2)^2 W_\phi^2 + c_2 W^4 - c_3 W^2 - \frac{n-2}{2\kappa^2} \Lambda_0, \quad (27)$$

where  $c_1 = -\frac{(n-4)}{2(n-2)}\beta\kappa^4$ ,  $c_2 = -\frac{(n-1)(n-4)}{8(n-2)^2}\beta\kappa^6$ , and  $c_3 = \frac{(n-1)}{2(n-2)}\kappa^2$ . Again, the parameter  $\beta$  has no effect on the Einstein equations in four-dimensional case. We will give the solutions of these equations for  $n = 4$  and  $n > 4$  with some choices of the superpotential, respectively.

The energy density  $\rho(y)$  of the system is given by  $\rho(y) = -T^0_0 = \frac{1}{2}\phi'^2 + V$ . For brane solutions, we require that the energy density on the boundaries of the extra dimension  $y$  vanishes:

$$\rho(|y| \rightarrow \infty) \rightarrow 0, \quad (28)$$

with which the naked cosmological constant  $\Lambda_0$  will be determined.

For  $n = 4$ ,  $c_1 = c_2 = 0$ . In order for the scalar to get a kink solution, the potential  $V(\phi)$  should at least has two finite vacua. And the usual  $\phi^4$  potential is a natural

choice. However, with the superpotential method, the  $\phi^4$  potential derived from the superpotential  $W(\phi) = a\phi + b\phi^2$  can not support kink solution for the scalar. This is because the coefficient of the  $\phi^4$  term is negative, which results in that there is only one finite vacuum in the scalar potential. Hence, we turn to use another superpotential  $W(\phi) = a\left(\phi - \frac{\phi^3}{3v_0^2}\right)$ , which yields the  $\phi^6$  model:

$$V(\phi) = -\frac{a^2\kappa^2}{12v_0^4} (\phi^2 - v_0^2)^2 [\phi^2 - 2(3\kappa^{-2} + 2v_0^2)], \quad (29)$$

where the two vacua are at  $\phi_\pm = \pm v_0$  (the extreme points of the superpotential  $W(\phi)$ ), and the naked cosmological constant  $\Lambda_0 = -\frac{1}{3}a^2v_0^2\kappa^4$ . The solution is found to be

$$\phi(y) = v_0 \tanh(ky), \quad (n = 4) \quad (30)$$

$$e^{2A(y)} = [\cosh(ky)]^{-\frac{2}{3}\kappa^2 v_0^2} e^{-\frac{1}{6}\kappa^2 v_0^2 \tanh^2(ky)}, \quad (31)$$

where  $k = a/v_0$ , and  $k > 0$  and  $k < 0$  correspond to kink and anti-kink solutions, respectively.

The energy density  $\rho(y)$  of the system is

$$\rho(y) = a^2 \left[ \frac{\kappa^2 v_0^2}{12} (3 + \text{sech}^2(ky)) + 1 \right] \text{sech}^4(ky). \quad (32)$$

The scalar curvature  $R(y)$  has a similar expression to the above one. At the boundaries of the extra dimension, i.e.,  $y \rightarrow \pm\infty$ , the energy density will vanish:  $\rho(|y| \rightarrow \infty) \propto e^{-4k|y|} \rightarrow 0$ . And the bulk curvature  $R(|y| \rightarrow \infty) \rightarrow -\frac{4}{3}a^2v_0^2\kappa^4 = 4\Lambda_0$ , which means that the spacetime is asymptotically anti-de Sitter. The corresponding cosmological constant is just the naked one:  $\Lambda = \Lambda_0$ . As shown in fig. 1, this solution describes a typical thick braneworld embedded in an AdS spacetime.

For general  $n > 4$  and  $\beta \neq 0$ , we have chance to get the usual  $\phi^4$  potential by setting  $W = a\phi$ . The potential is

$$V(\phi) = b(\phi^2 - v_0^2)^2, \quad (33)$$

where

$$b = \frac{n-4}{8(n-2)^2} [(n-4)a^2\kappa^2\beta^2 - (n-1)\beta] a^4\kappa^6, \\ v_0^2 = -\frac{2(n-2)}{(n-4)a^2\kappa^4\beta},$$

and the corresponding naked cosmological constant is

$$\Lambda_0 = \frac{n-1}{(n-2)(n-4)\beta}, \quad (34)$$

When  $\beta > 0$ , the above  $V(\phi)$  (33) is not a usual  $\phi^4$  potential with two degenerate vacua since  $v_0^2 < 0$ . Such potential does not support a thick brane solution because the energy density is divergent at the boundaries of the extra dimension  $y$ .

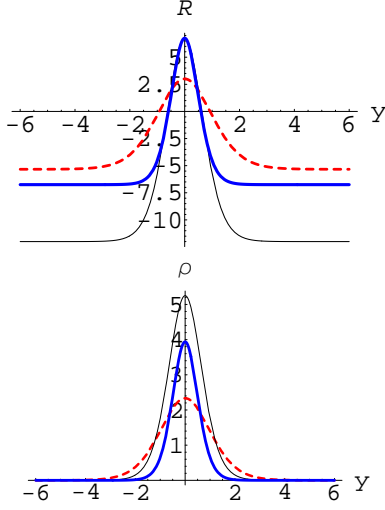


FIG. 1: The shapes of the scalar curvature  $R(y)$  and the energy density  $\rho(y)$  for the case  $n = 4$ . The parameters are set to  $\kappa = 1$ ,  $a = 1$  and  $v_0 = 2$  for red dashed lines,  $a = 1.5$  and  $v_0 = 2$  for black thin lines, and  $a = 1.5$  and  $v_0 = 1.5$  for blue thick lines.

So we are only interested in the case of  $\beta < 0$ , for which  $v_0^2 > 0$ ,  $b > 0$ , and the potential  $V(\phi)$  has two vacua at  $\phi_{\pm} = \pm v_0$ . The solution is

$$\phi(y) = v_0 \tanh(ky), \quad (n > 4, \beta < 0) \quad (35)$$

$$e^{2A(y)} = [\cosh(ky)]^{-\frac{2}{n-2}\kappa^2 v_0^2}, \quad (36)$$

where  $k = a/v_0 = \sqrt{-\frac{(n-4)}{2(n-2)}\kappa^4\beta}$ . This solution stands for a thick flat brane with the energy density given by

$$\rho(y) = \frac{1}{2}v_0^2 (k^2 + 2bv_0^2) \text{sech}^4(ky). \quad (37)$$

The thickness of the brane is of about  $1/k$ . On the boundaries  $|y| \rightarrow \infty$ , the solution of the warp factor is

$$A(|y| \rightarrow \infty) \rightarrow -\sqrt{\frac{-2\Lambda_0}{n-1}} |y|. \quad (38)$$

Note that the asymptotic solution (38) with the relation (34) is in accord with the thin brane solution (18)-(19) given in previous section. From the asymptotic solution (38), we have  $R_{MN}(|y| \rightarrow \infty) \rightarrow 2\Lambda_0 g_{MN} = \Lambda g_{MN}$ . Therefore, the thick flat brane is embedded in an asymptotically AdS spacetime with the cosmological constant  $\Lambda = 2\Lambda_0$ . In  $n(> 4)$ -dimensional critical gravity without matter fields, there are two disconnect AdS vacua with the cosmological constants determined by  $\Lambda_0 = \Lambda - \frac{(n-2)(n-4)\beta}{4(n-1)}\Lambda^2$  [5]. In our case here, because of the relation (34), which is caused by the condition (28), the

two asymptotic AdS vacua become the same one with the cosmological constant  $\Lambda = 2\Lambda_0$ .

In summary, we have studied the recently-proposed critical gravity theory [5], but the matter part has been introduced (namely the non-dS and non-AdS vacua) here. We considered the static flat brane world solutions in thin and thick braneworld scenarios, respectively. We found that the equations of motion for the braneworld scenarios are fourth-order if the critical condition is not introduced, hence in this case there are no thin brane solutions [28]. However, in critical case, the EOMs are two-order and the thin and thick brane solutions in arbitrary dimension are obtained. It was also found that the curvature-squared modifications in the four-dimensional critical gravity do not affect the brane solutions, but they will do in higher dimensions. All these branes are embedded in higher-dimensional AdS spacetimes.

For thick brane scenario, because the scalar  $\phi$  has a kink solution, fermions can be localized on the thick branes by introducing the Yukawa coupling  $\eta\bar{\Psi}\phi\Psi$  (see e.g. [15–19]).

Now, we compare our thick brane solutions given in this paper with the one in  $f(R)$  gravity with  $f(R) = R + \alpha R^2$  [20] (some other  $f(R)$ -brane solutions were considered in [21–27]). The action is

$$S = \int d^5x \sqrt{-g} \left[ \frac{1}{2\kappa^2} (R - 3\Lambda_0 + \alpha R^2) - \frac{1}{2} (\partial\phi)^2 - V(\phi) \right]. \quad (39)$$

The thick brane is also generated by a scalar field with the usual  $\phi^4$  potential. The equations of motion are four-order in this  $R^2$  gravity, and the solution is given by [20]

$$\phi(y) = v_0 \tanh(ky), \quad (40)$$

$$e^{2A(y)} = \cosh^{-2}(ky), \quad (41)$$

$$\Lambda_0 = -\frac{159}{3364\alpha}, \quad (42)$$

where  $k = \sqrt{\frac{3}{232\alpha}}$ . It was shown that the linear tensor perturbation equations of the brane metric are two-order. The perturbations are stable and gravity can be localized on the brane [20, 26]. While, for the scenario of critical gravity, although the equations of motion for brane model are two-order, the linear perturbation equations of the metric are four-order. So, there are some important questions need to answer: Are those braneworlds in critical gravity stable under linear metric perturbations? Can gravitons be localized on the branes? We would like to investigate these problems in future.

We are grateful to Prof. H. Lü for useful discussions. This work was supported by the Program for New Century Excellent Talents in University, the National Natural Science Foundation of China (No. 11075065), and the Fundamental Research Funds for the Central Universities (No. lzujbky-2012-k30).

- 
- [1] K. S. Stelle, Phys. Rev. **D16** (1977) 953.
  - [2] K. S. Stelle, Gen. Rel. Grav. **99** (1978) 353.
  - [3] W. Li, W. Song and A. Strominger, JHEP **0804** (2008) 082.
  - [4] E. A. Bergshoeff, O. Hohm and P. K. Townsend, Phys. Rev. Lett. **102** (2009) 201301.
  - [5] H. Lu and C. N. Pope, Phys. Rev. Lett. **106** (2011) 181302.
  - [6] S. Deser, H. Liu, H. Lu, C. N. Pope, T. C. Sisman and B. Tekin, Phys. Rev. **D83** (2011) 061502.
  - [7] D. Grumiller and N. Johansson, JHEP **0807** (2008) 134.
  - [8] M. Alishahiha and R. Fareghbal, Phys. Rev. **D83** (2011) 084052.
  - [9] E. A. Bergshoeff, O. Hohm, J. Rosseel and P. K. Townsend, Phys. Rev. **D83** (2011) 104038.
  - [10] M. Porrati and M. M. Roberts, Phys. Rev. **D84** (2011) 024013.
  - [11] N. Ohta, *A complete classification of higher derivative gravity in 3D and criticality in 4D*, Class. Quant. Grav. **29** (2012) 015002.
  - [12] H. Liu, H. Lu and M. Luo, *On black hole stability in critical gravities*, arXiv:1104.2623[hep-th].
  - [13] L. Randall and R. Sundrum, Phys. Rev. Lett. **83** (1999) 3370.
  - [14] L. Randall and R. Sundrum, Phys. Rev. Lett. **83** (1999) 4690.
  - [15] R. Koley and S. Kar, Class. Quant. Grav. **22** (2005) 753.
  - [16] A. Melfo, N. Pantoja and J. D. Tempo, Phys. Rev. **D 73** (2006) 044033.
  - [17] T. R. Slatyer and R. R. Volkas, JHEP **0704** (2007) 062.
  - [18] Y.-X. Liu, C.-E. Fu, L. Zhao and Y.-S. Duan, Phys. Rev. **D80** (2009) 065020.
  - [19] C. A. S. Almeida, M. M. Ferreira, Jr., A. R. Gomes, and R. Casana, Phys. Rev. **D 79** (2009) 125022.
  - [20] Y.-X. Liu, Y. Zhong, Z.-H. Zhao and H.-T. Li, JHEP **1106** (2011) 135.
  - [21] C. Charmousis, S. C. Davis, and J. F. Dufaux, JHEP **12** (2003) 029.
  - [22] M. Parry, S. Pichler, and D. Deeg, *Higher-derivative gravity in brane world models*, JCAP **0504** (2005) 014.
  - [23] V. I. Afonso, D. Bazeia, R. Menezes, and A. Y. Petrov, Phys. Lett. **B 658** (2007) 71.
  - [24] V. Dzhunushaliev, V. Folomeev, B. Kleihaus, and J. Kunz, *Some thick brane solutions in  $f(R)$ -gravity*, JHEP **04** (2010) 130.
  - [25] J. Hoff da Silva and M. Dias, Phys. Rev. **D 84** (2011) 066011.
  - [26] Y. Zhong, Y.-X. Liu, and K. Yang, Phys. Lett. **B 699** (2011) 398.
  - [27] H. Liu, H. Lu and Z.-L. Wang,  *$f(R)$  gravities, Killing spinor equations, “BPS” domain walls and cosmology*, arXiv:1111.6602[hep-th].
  - [28] This conclusion is obtained directly by observing the Einstein equations (5). In order to embed  $(n-2)$ -branes, one usually ask  $A'''' \sim \delta(y)$ , so that  $A'''$  contains a skip, while  $A''$ ,  $A'$  and  $A$  are continuous. But the non-continuous skipping function  $A'''$  brings some troubles. Recall that in RS brane world model, the step function  $A' \sim \epsilon(y)$  appears in the equation of motion in terms of  $A'^2$ , which is continuous. However, in critical gravity, we get a non-continuous term  $A'''A'$ , which cannot be canceled by other terms at the skipping point. Thus the system we considered in (1)-(4) supports no thin brane solution if without the critical condition.